Corporate Finance (FNCE 611/612)

PLACEMENT/WAIVER EXAM–PART 1

Instructions

1. **Please don't open the exam until you are told to do so.**

2. This exam is being administered under the University's rules for academic conduct; the Code of Academic Integrity applies.

3. The exam consists of 5 multiple choice questions and 4 essay questions.

4. Use the white spaces (and backs of pages) in this question booklet as scratch paper for the multiple choice questions. Your final answers should be indicated **with a pen** in the appropriate boxes on page 1 of your answer sheet booklet.

5. Write your answers to the essay questions in the answer sheet booklet. You can use a pen or a pencil. If you need more answer sheets, you can request them, and they will be provided for you. *Do not use the backs of answer sheets, since these back pages will not be graded.*

6. IMPORTANT: Print your name and Penn ID number on the first page of your answer sheet booklet.

7. This is an open-book exam, i.e. you are free to use any course material. You are allowed to use a pocket calculator. Laptop computers and cell phones are not allowed.

8. You have two hours. The time left in the exam will be announced periodically. If you finish early, you can quietly hand in your answer sheet booklet and leave, *unless there is less than ten minutes left in the exam.*

9. **Please stop writing when requested to. There will be a penalty of 20 points for the people who don't.**

10. Remain seated until all the answer sheet booklets (not just yours) have been collected.

PART I: MULTIPLE CHOICE QUESTIONS *(Total points: 25)*

*INSTRUCTIONS: A correct answer to each of these questions is worth 5 points. An incorrect answer is worth 0. Also, for each question that you choose not to answer, you get 1 point. If you do choose to answer, write your answer **clearly** in the appropriate box on page 1 of your Answer Sheet Booklet. Your answers should be **capital** letters written with a **pen**. An empty box will be interpreted as a "no answer."*

1. A 10-year annuity paying $\$x$ at the beginning of every year (i.e. the first of ten payments is made today) is worth the same (today) as an annuity of $300 payable every 6 months for 10 years (20 payments), the first payment of which is due 66 months from now. If the annual interest rate (compounded annually) is 3%, find x.

a. $232.73 **d.** $508.11

b. $502.48 **e.** $521.42

c. $506.23

2. A machine costing $3,000 must be replaced at the end of 8 years. The resale value of the machine at the time of replacement is $600. At what annual discount rate (compounded annually) would it be equally economical to use a similar machine costing $4,000 with a life of 8 years and a resale value of $1,900? (Assume that there is no taxes.)

a. 2.4% **d.** 3.3%

b. 2.7% **e.** 3.6%

c. 3.0%

3. What is the present value of 15 payments of $100 each received every 18 months (the first one occuring in 18 months from now), if the annual discount rate (compounded annually) is 9%?

a. $620.43 **d.** $951.28

b. $875.56 **e.** $1,209.10

c. $930.61

4. Corporate managers can maximize shareholder wealth by choosing positive NPV projects because:

a. all investors have the same preferences.

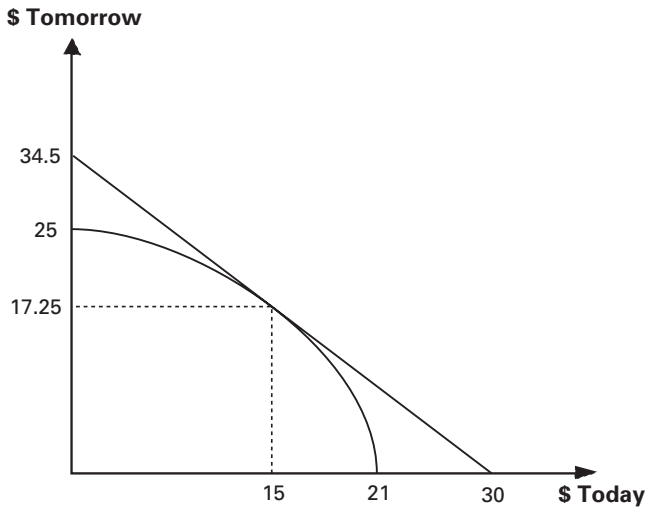
b. the unhappy shareholders can sell off their shares.

c. given the existence of financial markets, investors will be satisfied with the same real investment decisions regardless of personal preferences.

d. managers are wiser than shareholders regarding investments.

e. none of the above.

5. In the figure below, the sloping straight line represents the opportunities for investment in the capital market, and the solid curved line represents the opportunities for investment in plant and machinery (real assets). The company's only asset at present is $21 million in cash.



Note that the figure is not drawn to scale, and that all the numbers are in millions.

Let I denote the optimal amount that should be invested in real assets, and r the interest rate in capital markets. Calculate I/r.

a. 3.2 million
b. 12 million
c. 32 million
d. 40 million
e. 60 million

PART II: ESSAY QUESTIONS *(Total points: 75)*

INSTRUCTIONS: Each of the following questions is to be answered in the Answer Sheet Booklet. You can use a pen or a pencil. If you need more answer sheets, you can request them, and they will be provided for you. ***Do not use the backs of answer sheets****, since these back pages will not be graded. The number of points for each question is indicated in parentheses at the beginning of the question. In answering these questions, make sure to show all your calculations; in particular,* ***no points will be given for calculator shortcuts.***

1. *(20 points)* Every year, you receive your entire annual salary at the end of the year. This year, your end-of-year salary will be $50,000 (in nominal terms). In real terms, you expect your salary to increase at a rate of 2% per year in the future.

You have decided to start saving for retirement by putting money in a savings account. You plan to retire in 35 years, and you expect to live for 25 years after that. You assess that a reasonable lifestyle during those 25 years will require you to have, at the end of every year, a disposable income of $25,000 in real terms (i.e. the same purchasing power as $25,000 today). The nominal interest rate on your savings account is 8%, and it is expected to stay at that rate forever. The real interest rate is also expected to stay at its current level of 3.5%.

a. What is the inflation rate?

b. How much money (in nominal terms) will you need to have in your savings account when you retire, in 35 years (end of year 35), in order to be able to enjoy the lifestyle that you find reasonable? *HINT: First calculate the amount that you will need in real terms.*

c. Suppose that you will start saving for retirement at the end of the current year. Suppose further that you plan to make 35 deposits (one at the end of every year). All deposits are a fixed fraction x of your salary. Find the fraction x that will allow you to reach your "reasonable lifestyle" objective. *HINT: You will need to make use of the growing annuity formula.*

2. *(15 points)* You are a financial analyst for a company that is considering a new project. If the project is accepted, it will use a fraction of a storage facility that the company already owns but currently does not use. The project is expected to last 10 years, and the annual discount rate is 10% (compounded annually).

You research the possibilities, and find that the entire storage facility can be sold for $100,000 and a smaller (but big enough) facility can be acquired for $40,000. The book value of the existing facility is $60,000, and both the exisiting and the new facilities (if it is acquired) would be depreciated straight line over 10 years (down to a zero book value). The corporate tax rate is 40%. What is the opportunity cost of using the existing storage capacity? *HINT: Think about what you would gain and lose if you did not.*

3. *(15 points)* You own a rental building in the city and are interested in replacing the heating system. You are faced with the following alternatives:

a. A solar system, which will cost $12,000 to install and $500 at the end of every year to run, and will last forever (assume that your building will too).

b. A gas-heating system, which will cost $5,000 to install and $1,000 at the end of every year to run, and will last 20 years.

c. An oil-heating system, which will cost $3,500 to install and $1,200 at the end of every year to run, and will last 15 years.

If your opportunity cost of capital (discount rate) is 10%, which of these three options is best for you?

4. *(25 points)* The following bonds are traded in a well functioning market:

BOND	TYPE	FACE VALUE	COUPON	MATURITY	PRICE
A	Zero Coupon Bond	$100	—	1 year	$92.00
B	Coupon Bond	$100	8%	2 years	$101.32

a. Assuming that the coupon bond (bond B) makes only annual payments, what discount factors (DF_1, DF_2) are imbedded in these prices? *NOTE: Show all your calculations; no points will be given for answers found by a sophisticated calculator.*

b. What are the 1-year, and 2-year spot rates (r_1 and r_2)?

c. Suppose that you would like to purchase a two-year coupon bond with a face value of $10,000 and a coupon rate of 6% (with annual coupon payments). Since such a bond is not traded in this economy, what portfolio of bonds A and B could you form to satisfy your needs (i.e. how can you replicate this bond using the original two bonds). *NOTE: Make sure to describe that portfolio clearly, i.e. what you are buying/selling.*

d. What is the exact yield to maturity on

i. bond A;

ii. bond B.

NOTE: Again, show all your calculations; no points will be given for answers found by a sophisticated calculator. In particular, you will need to use the following formula for the roots of $ax^2 + bx + c = 0$*:*

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Your answers should have at least two decimals, like 9.53%.

Corporate Finance (FNCE 611/612)

PLACEMENT/WAIVER EXAM–PART 2

Instructions

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3. The exam consists of 5 multiple choice questions and 4 essay questions.

4. Write all of your answers in the blue booklets. If you need more booklets, you can request them, and they will be provided for you.

5. You must cross out anything that you do not wish to have marked. For the multiple choice questions, please write your letter answers **with a pen**. (You can do your calculations in either pen or pencil. The calculations will not be marked, only the letter answer, so no partial credit is given). For the essay questions, you may use either a pen or a pencil; partial credit may be given.

6. **Important:** Print your name and Penn ID number on the first page of your answer sheet booklet. Also indicate which section of the course you attend, so I can return your exam in the proper section.

7. This is an open-book exam, i.e., you are free to use any course material. You are allowed to use a pocket calculator. Laptop computers and cell phones are not allowed.

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PART I: MULTIPLE CHOICE QUESTIONS *(Total points: 25)*

*INSTRUCTIONS: A correct answer to each of these questions is worth 5 points. An incorrect answer is worth 0. Also, for each question that you choose not to answer, you get 1 point. If you do choose to answer, write your answer **clearly** on page 1 of your blue booklet. Your answers should be **capital** letters written with a **pen**. Only the final answer will be marked and there shall be no partial credit for the multiple choice questions.*

1. *(5 points)* Suppose that the price of the stock is S_0, and its annual volatility is σ. Suppose also that the annual riskfree rate is r_f. According to Black-Scholes, what is the price of a European put option with a strike price of X maturing in T years? NOTE: In the answers below, we use

$$x = \frac{\log\left(\frac{S_0}{X/(1+r_f)^T}\right)}{\sigma\sqrt{T}} + \frac{1}{2}\sigma\sqrt{T}.$$

a. $\dfrac{X}{(1+r_f)^T}\Big[1 - N(x - \sigma\sqrt{T})\Big] - S_0\Big[1 - N(x)\Big]$

b. $S_0\, N(x) - \dfrac{X}{(1+r_f)^T}\, N(x - \sigma\sqrt{T})$

c. $S_0\, N(x - \sigma\sqrt{T}) - \dfrac{X}{(1+r_f)^T}\, N(x)$

d. $S_0\Big[1 - N(x)\Big] - \dfrac{X}{(1+r_f)^T}\Big[1 - N(x - \sigma\sqrt{T})\Big]$

e. $S_0\Big[1 - N(x - \sigma\sqrt{T})\Big] - \dfrac{X}{(1+r_f)^T}\Big[1 - N(x)\Big]$

2. *(5 points)* Which of the following statements are true?

I. In a perfect capital market, it is advantageous for the firm to issue debt (vs. equity) to finance a project, because the cost of debt (r_D) is always smaller than the cost of equity (r_E).

II. The reason that Modigliani and Miller's Proposition I does not hold in the presence of corporate taxes is because levered firms pay less taxes than identical unlevered firms.

III. Equity financing is always better than debt financing when the personal tax rate on equity income (t_E) is smaller than the personal tax rate on interest income (t_D).

a. I and II

b. I and III

c. II and III

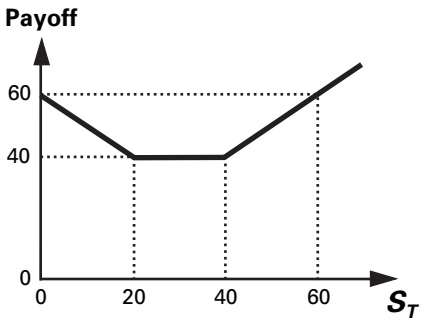
d. I, II and III

e. fewer than two statements are true.

3. *(5 points)* If a firm borrows $50 million for one year (i.e., the firm is levered for one year only) at an interest rate of 9%, what is the present value of the interest tax shield? Assume that there are no personal taxes, and that the corporate tax rate is 35%.

a. $50.000 million
b. $17.500 million
c. $4.128 million
d. $1.575 million
e. $1.445 million

4. *(5 points)* Suppose that you would like to take a position that will give you the following payoff at time T, as a function of the stock price S_T at that time:



Which of the following strategies will give you this position (assume that all the call and put options are European options maturing at T, and are written on the given stock)?

I. Buy 1 put with a strike price of 60, buy 1 call with a strike price of 40, and buy 1 call with a strike price of 20.

II. Buy 1 call with a strike price of 40, buy 1 put with a strike price of 20, and lend (at the riskfree rate) the present value of $40 deliverable at time T.

III. Buy 1 put with a strike price of 40, buy 1 put with a strike price of 20, and buy 1 share of the stock.

a. I and II
b. I and III
c. II and III
d. I, II and III
e. fewer than two positions will give you the desired payoff.

5. *(5 points)* The Hifalutin Corporation has no debt in its capital structure, and the expected rate of return on its equity is 15%. There are 300,000 shares outstanding. The company has expected annual pre-tax earnings of $3 million in perpetuity. The corporate tax rate is 40%. If Hifalutin announces that it will issue $3.75 million worth of perpetual debt and use the entire proceeds to buy back some stocks, what will be its new share price? (Ignore personal taxes.)

a. 40.00
b. 45.00
c. 48.50
d. 50.00
e. 52.50

PART II: ESSAY QUESTIONS *(Total points: 75)*

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1. *(20 points total)* Stiphla Inc.'s real assets are expected to generate earnings before interest and taxes (EBIT) of $102,000 at the end of every year in perpetuity. The firm is currently financed by 50,000 shares each worth $6.11 and by $130,000 worth of perpetual debt issued at a rate of 12%. The corporate tax rate is 35%. Ignore personal taxes and bankruptcy costs.

a. *(2 points)* What is the current total firm value of Stiphla Inc.?

b. *(3 points)* What is the current expected return on Stiphla's equity?

c. *(2 points)* What is Stiphla's weighted average cost of capital (WACC)?

d. *(2 points)* Show that the value of the firm can be obtained by discounting its after-tax earnings at the weighted average cost of capital.

Janine Finch, the CFO of the company, has just found out that Stiphla could issue an additional $130,000 worth of perpetual debt to buy back some equity. However, because the new debt will be junior to the original debt, Stiphla will have to pay a rate of 14% on that new debt.

e. *(2 points)* What is the value of the firm after it goes ahead with the new debt issue?

f. *(4 points)* What is the new expected return on the firm's equity?

g. *(2 points)* Explain why the shareholders are better off (in terms of their total wealth).

h. *(3 points)* What is Stiphla's new weighted average cost of capital (WACC)?

2. *(10 points total)* Firms A and B are both unlevered. The shares of both companies are currently trading at $100, and both offer an annual pre-tax return of 10%. In the case of firm A, the return is entirely in the form of a dividend yield (i.e., the company pays a regular annual dividend of $10 a share). In the case of firm B, the return comes entirely as capital gain (the shares appreciate by 10% a year). Suppose that an investor buys a share of each firm today, and plans to sell them in 10 years. Suppose that dividends and capital gains are both taxed at 30%.

a. *(5 points)* What is the annual after-tax yield (rate of return) on firm A's share over the 10-year period?

b. *(5 points)* What is the annual after-tax yield (rate of return) on firm B's share over the 10-year period?

3. *(25 points total)* The Jack & Diane (JD) Corporation is considering a new 5-year project. Since this project is very different from JD's current operations, the adjusted present value will be used to value the project.

The project requires an initial investment of $750,000 in new assets, which will be depreciated straight-line to 0 over the project's 5-year life. These assets will be worthless in five years, i.e., they will not be resold. (Assume that the depreciation tax shields can be discounted at the project discount rate). Each year for five years, the project is expected to generate pre-tax revenues of $600,000 and to require pre-tax costs of $240,000. The entire project will be financed through a 5-year bank loan with an annual rate of 10%. The principal on the loan will be repaid in equal installments of $150,000 each (i.e., each year, the company pays $150,000 in principal, and pays the interest on the outstanding loan). It is estimated that the pre-tax costs (payable at time zero) of negotiating the loan will be 4% of the amount borrowed.

The project's risk is very similar to the risk of Tommy & Gina (TG) Inc.'s assets. This firm is currently financed by 100,000 shares worth $12.50 each, and $750,000 worth of debt. The beta of TG's stock is 1.5, and the company borrows at a rate of 11%. The riskfree rate in the economy is 8%, and the expected return on the market is 18%. The current corporate tax rate is 45% (assume that it applies to both JD and TG). Ignore personal taxes.

a. *(8 points)* What would be the appropriate discount rate for the project, if it were all-equity financed?

b. *(17 points)* What is the adjusted present value of the project?

4. *(20 points total)* During the upcoming year, the stock price of Delinquent Jesters Inc. (DJ) is expected to go up to $290 or down to $170 with equal probabilities. The beta of the stock is equal to 0.75. The annual riskfree rate is 10.5%, and the expected annual return on the market is 16.5%. You are interested in replicating and pricing a European call option on DJ's stock. The option has a strike price of $212, and will mature in one year.

a. *(4 points)* What is the current price of DJ's stock?

b. *(6 points)* Using the stock and borrowing/lending (at the riskfree rate), form a portfolio that will replicate the call option. How many shares of DJ will you buy/sell, and how much money will you borrow/lend?

c. *(3 points)* Use the portfolio derived in part (b) to price the call option.

d. *(4 points)* What is the beta and expected return of the call option?

e. *(3 points)* Using the result from part (d), show that the price of the call option found in part (c) can also be derived by discounting the expected cash flow of the option.

Placement/Waiver Exam-Part 1 Answers FNCE 611/612

PART I: Multiple Choice Questions

1. **C**. We must have

$$x\,\ddot{a}_{\overline{10}|3\%} = \frac{300}{(1.03)^5} a_{\overline{20}|\hat{r}},$$

where the equivalent semiannual interest rate $\hat{r}$ must satisfy

$$(1+\hat{r})^2 = 1.03 \quad \Rightarrow \quad \hat{r} = 1.4889157\%.$$

Since $\ddot{a}_{\overline{10}|3\%} = 8.7661089$ and $a_{\overline{20}|\hat{r}} = 17.1874132$, we find $x = 506.23$.

2. **D**. The interest rate r must satisfy:

$$\begin{aligned} & -3{,}000 + \frac{600}{(1+r)^8} = -4{,}000 + \frac{1{,}900}{(1+r)^8} \\ \Leftrightarrow \quad & 1{,}000 = \frac{1{,}300}{(1+r)^8} \\ \Leftrightarrow \quad & r = \left(\frac{1{,}300}{1{,}000}\right)^{1/8} - 1 = 3.33\%. \end{aligned}$$

3. **A**. First, let us find the equivalent 18-month rate $\hat{r}$:

$$(1.09)^{3/2} = 1 + \hat{r} \quad \Rightarrow \quad \hat{r} = 13.79934\%.$$

The present value PV of the annuity is therefore

$$PV = 100 a_{\overline{15}|\hat{r}} = \frac{100}{0.1379934}\left[1 - \frac{1}{(1.1379934)^{15}}\right] = 620.43.$$

4. **C**. See section I.3.1 of the lecture notes.

5. **D**. The amount invested in real assets is given by

$$I = 21 \text{ million} - 15 \text{ million} = 6 \text{ million}.$$

The slope of the straight line (capital market investment opportunities) is $-(1+r)$, that is

$$-(1+r) = -\frac{34.5}{30} \quad \Rightarrow \quad r = 15\%.$$

Therefore, $I/r = 40$ million.

PART II: Essay Questions

1. (a) (4 points) The inflation rate is given by

$$i = \frac{1+r}{1+R} - 1 = \frac{1.08}{1.035} - 1 = 4.3478261\%.$$

(b) (8 points) At the end of 35 year, the present value PV_R in real terms of your retirement income is

$$PV_R = 25{,}000 a_{\overline{25}|3.5\%} = \frac{25{,}000}{0.035}\left[1 - \frac{1}{(1.035)^{25}}\right] = 412{,}037.86.$$

Since this amount is in real terms, we need to inflate it for 35 years. Therefore, the nominal amount PV_n needed in the account in 35 years is

$$PV_n = 412{,}037.86(1.043478261)^{35} = 1{,}827{,}495.55.$$

(c) (8 points) The present value at time 0 of the amount needed in the account in 35 years is

$$PV_0 = \frac{412{,}037.86}{(1.035)^{35}} = 123{,}601.83.$$

Alternatively, we could do the calculations in nominal terms:

$$PV_0 = \frac{1{,}827{,}495.55}{(1.08)^{35}} = 123{,}601.83.$$

The present value of your 35 contributions should be equal to this amount. In real terms:

$$123{,}601.83 = \frac{50{,}000x/(1.043478261)}{0.035 - 0.02}\left[1 - \left(\frac{1.02}{1.035}\right)^{35}\right] \quad \Rightarrow \quad x = 9.6713\%.$$

Again, the calculations could have been done in nominal terms, in which case they grow at $g = (1.02)(1+i) - 1 = 6.4347826\%$:

$$123{,}601.83 = \frac{50{,}000x}{0.08 - 0.064347826}\left[1 - \left(\frac{1.064347826}{1.08}\right)^{35}\right] \quad \Rightarrow \quad x = 9.6713\%.$$

2. (15 points) By selling the existing facility, the company would

- gain \$100,000 from the sale;
- pay a tax of
$$(\$100{,}000 - \$60{,}000)(40\%) = \$16{,}000$$
on the capital gain resulting from this sale;
- lose the yearly depreciation tax shield of
$$\frac{\$60{,}000}{10}(40\%) = \$2{,}400$$
for 10 years.

By acquiring the new facility, the company would

- pay \$40,000 to buy the facility;
- gain a yearly depreciation tax shield of

$$\frac{\$40{,}000}{10}(40\%) = \$1{,}600$$

for 10 years.

The present value PV of all these gains and losses represents the opportunity cost of using the existing storage capacity:

$$\begin{aligned} PV &= 100{,}000 - 16{,}000 - 2{,}400a_{\overline{10}|10\%} - 40{,}000 + 1{,}600a_{\overline{10}|10\%} \\ &= 100{,}000 - 16{,}000 - \frac{2{,}400}{0.10}\left[1 - \frac{1}{(1.10)^{10}}\right] - 40{,}000 + \frac{1{,}600}{0.10}\left[1 - \frac{1}{(1.10)^{10}}\right] \\ &= 39{,}084.35. \end{aligned}$$

3. (15 points) There are two equivalent approaches for solving this problem: (i) repeat the cash flows to infinity (which is already done for alternative A), and calculate and compare the net present values; (ii) Calculate and compare the equivalent annual costs of the three alternatives. Let us use the second approach.

   A. The present value of the costs is

$$PV_A = 12{,}000 + \frac{500}{0.10} = 17{,}000.$$

   The equivalent annual cost C_A must solve

$$17{,}000 = \frac{C_A}{0.10} \quad \Rightarrow C_A = 1{,}700.$$

   B. The present value of the costs is

$$PV_B = 5{,}000 + \frac{1{,}000}{0.10}\left[1 - \frac{1}{(1.10)^{20}}\right] = 13{,}513.56.$$

   The equivalent annual cost C_B must solve

$$13{,}513.56 = \frac{C_B}{0.10}\left[1 - \frac{1}{(1.10)^{20}}\right] \quad \Rightarrow C_B = 1{,}587.30.$$

   C. The present value of the costs is

$$PV_C = 3{,}500 + \frac{1{,}200}{0.10}\left[1 - \frac{1}{(1.10)^{15}}\right] = 12{,}627.30.$$

   The equivalent annual cost C_C must solve

$$12{,}627.30 = \frac{C_C}{0.10}\left[1 - \frac{1}{(1.10)^{15}}\right] \quad \Rightarrow C_C = 1{,}660.16.$$

Therefore, alternative B is the best alternative, since it involves the lowest costs.

4. (a) (6 points) Since the price of every bond must be the sum of its discounted cash flows, the discount factors must solve:

$$\begin{aligned} 100DF_1 &+ && = 92.00 \quad (1)\\ 8DF_1 &+ 108DF_2 &= 101.32 \quad (2)\end{aligned}$$

Using (1), we have $DF_1 = 0.92$. Using this value for DF_1 in (2), we get

$$DF_2 = \frac{101.32 - 8(0.92)}{108} = 0.87.$$

(b) (4 points) The discount factors can be written as

$$DF_t = \frac{1}{(1+r_t)^t}.$$

Therefore,

$$r_1 = \frac{1}{DF_1} - 1 = 8.69565\%, \quad \text{and}$$
$$r_2 = \frac{1}{DF_2^{1/2}} - 1 = 7.21125\%.$$

(c) (7 points) The bond that you would like to purchase will pay 6%($10,000) = $600 at the end of the first year, and $10,600 at the end of the second year. Let us form a portfolio containing a quantity n_A of bond A, and n_B of bond B. We would like this portfolio to pay $600 at the end of the first year, and $10,600 at the end of the second year. Mathematically we would like n_A and n_B to satisfy:

$$\begin{aligned} 100\ n_A &+ 8\ n_B &= 600 \quad (1)\\ &+ 108\ n_B &= 10{,}600 \quad (2)\end{aligned}$$

Using (2), we have $n_B = \frac{10,600}{108} = 98.148148$. Using this value for n_B in (1), we get

$$n_A = \frac{600 - 8(98.148148)}{100} = -1.851852.$$

Therefore, the portfolio that would replicate the 6% coupon bond would consist in selling 1.851852 units of bond A, and buying 98.148148 units of bond B.

(d) (i) (3 points) The yield on a zero-coupon bond with a maturity of t years is simply the t-year spot rate. Therefore the yield y_A of bond A is $y_A = r_1 = 8.69565\%$.

(ii) (5 points) The yield to maturity y_B for bond B has to satisfy

$$101.32 = \frac{8}{1+y_B} + \frac{108}{(1+y_B)^2} \quad \Leftrightarrow \quad 108x^2 + 8x - 101.32 = 0, \text{ where } x = \frac{1}{1+y_B}.$$

Solving for x using the quadratic equation formula, we find

$$\frac{1}{1+y_B} = x = \frac{-8 \pm \sqrt{(8)^2 - 4(108)(-101.32)}}{2(108)} = 0.9327379.$$

Solving for y_B (ignoring the "minus" root, which has no economic meaning), we find $y_B = 7.26721\%$.

Placement/Waiver Exam-Part 2 Answers FNCE 611/612

PART I: Multiple Choice Questions

1. **A**. We know from Black-Scholes that the price of a call option with strike X maturity T is

$$C_0 = S_0\,N(x) - \frac{X}{(1+r_f)^T}\,N(x-\sigma\sqrt{T}).$$

We can then find the price of the put by using the put-call parity relationship:

$$\begin{aligned} P_0 &= C_0 - S_0 + \frac{X}{(1+r_f)^T} \\ &= S_0\,N(x) - \frac{X}{(1+r_f)^T}\,N(x-\sigma\sqrt{T}) - S_0 + \frac{X}{(1+r_f)^T} \\ &= \frac{X}{(1+r_f)^T}\Big[1-N(x-\sigma\sqrt{T})\Big] - S_0\Big[1-N(x)\Big]. \end{aligned}$$

2. **E**. (I) FALSE. In a perfect capital market, the capital structure of a firm does not affect its value. (II) TRUE. The advantage of debt financing comes from the very fact that less taxes are being paid. (III) FALSE. Equity financing is better if $(1-t_E)(1-t_c) > (1-t_D)$. This is not always the case when $t_E < t_D$: if t_c is large enough, the inequality goes the other way, and debt financing is more advantageous.

3. **E**. The interest payment at the end of the year is $0.09 \times 50 = 4.5$. This amount is expected to shield $0.35 \times 4.5 = 1.575$ of the firm's profits from taxes. The present value of this amount is $1.575/1.09 = 1.445$.

4. **C**. It can be shown graphically that the last two strategies will give you the desired position. The first one is incorrect for $S_T > 60$ (the slope after that point is 2, not 1, as desired).

5. **B**. The value of the unlevered company (before the debt issue) is

$$V_U = \frac{(1-0.4)3{,}000{,}000}{0.15} = 12{,}000{,}000.$$

The value of the levered company (after the debt issue) will be

$$V_L = V_U + t_c D = 12{,}000{,}000 + 0.4(3{,}750{,}000) = 13{,}500{,}000.$$

The new equity value will be

$$E = V_L - D = 13{,}500{,}000 - 3{,}750{,}000 = 9{,}750{,}000.$$

Letting n denote the number of shares repurchased and S the new price per share, we must have

$$\begin{aligned} nS &= 3{,}750{,}000 \\ (300{,}000 - n)S &= 9{,}750{,}000 \end{aligned}$$

Solving for n and S gives $n = 83{,}333.33$ and $S = 45$.

PART II: Essay Questions

1. (20 points total)

   (a) (2 points) We have $D_L = 130{,}000$ and $E_L = 50{,}000 \times 6.11 = 305{,}500$, so that $V_L = D_L + E_L = 435{,}500$.

   (b) (3 points) The value of the equity can be obtained by discounting the after-tax earnings that will be received by the shareholders:

$$E_L = \frac{(1 - t_c)(\text{EBIT} - r_D D_L)}{r_E}.$$

   This implies

$$\begin{aligned} r_E &= \frac{(1 - t_c)(\text{EBIT} - r_D D_L)}{E_L} \\ &= \frac{(1 - 0.35)[102{,}000 - (0.12)(130{,}000)]}{305{,}500} \\ &= 18.38298\%. \end{aligned}$$

   (c) (2 points) Stiphla's weighted average cost of capital is

$$\begin{aligned} \text{WACC} &= (1 - t_c) r_D \frac{D_L}{V_L} + r_E \frac{E_L}{V_L} \\ &= (1 - 0.35)(0.12)\frac{130{,}000}{435{,}500} + (0.1838298)\frac{305{,}500}{435{,}500} \\ &= 15.22388\%. \end{aligned}$$

   (d) (2 points) The value of the firm can also be obtained as follows:

$$V_L = \frac{(1 - t_c)\text{EBIT}}{\text{WACC}} = \frac{(1 - 0.35)102{,}000}{0.1522388} = 435{,}500.$$

   (e) (2 points) Let us denote by primed variables all the quantities after the new debt issue. The firm's value will go up by the present value of its additional tax shields:

$$V_L' = V_L + t_c(D_L' - D_L) = 435{,}500 + 0.35(260{,}000 - 130{,}000) = 481{,}000.$$

(f) (4 points) The equity is now worth

$$E'_L = V'_L - D'_L = 481{,}000 - 260{,}000 = 221{,}000.$$

As before, the value of the equity can be obtained by discounting the after-tax earnings that will be received by the shareholders:

$$E'_L = \frac{(1-t_c)[\text{EBIT} - (0.12)(130{,}000) - (0.14)(130{,}000)]}{r'_E}.$$

This implies

$$\begin{aligned} r'_E &= \frac{(1-t_c)[\text{EBIT} - (0.12)(130{,}000) - (0.14)(130{,}000)]}{E'_L} \\ &= \frac{(1-0.35)[102{,}000 - (0.12)(130{,}000) - (0.14)(130{,}000)]}{221{,}000} \\ &= 20.05882\%. \end{aligned}$$

(g) (2 points) The shareholders are better off because their wealth went from \$305,500 (in equity only) to \$351,000 (\$221,000 in equity, and \$130,000 in cash from the debt issue).

(h) (3 points) Stiphla's new weighted average cost of capital will be given by

$$\text{WACC}' = (1-t_c)(12\%)\frac{130{,}000}{481{,}000} + (1-t_c)(14\%)\frac{130{,}000}{481{,}000} + r'_E\frac{221{,}000}{481{,}000} = 13.78378\%.$$

Equivalently, the total debt of \$260,000 is issued at an average rate of $\frac{12\%+14\%}{2} = 13\%$, so that

$$\text{WACC}' = (1-t_c)(13\%)\frac{260{,}000}{481{,}000} + r'_E\frac{221{,}000}{481{,}000} = 13.78378\%.$$

Finally, the firm's weighted average cost of capital can also be found by using the fact that the firm's value is given by the after-tax earnings discounted at the WACC:

$$V'_L = \frac{(1-t_c)\text{EBIT}}{\text{WACC}'} \quad \Leftrightarrow \quad \text{WACC}' = \frac{(1-0.35)102{,}000}{481{,}000} = 13.78378\%.$$

(10 points total)

(a) (5 points) Every year, the investor receives \$10, which is taxed at 30%. So, after taxes, the investor receives \$7 each year. At the end of year 10, the investor will sell his share of firm A for \$100, and so will not have any capital gain. His annual after-tax return on his \$100, is therefore $7/100 = 7\%$.

(b) (5 points) The investor will not receive any money until year 10, at which point he will sell his share of firm B for $100(1.10)^{10} = 259.37$. The capital gains of $259.37 - 100 = 159.37$ will then be taxed at 30%. Therefore, the annual after-tax rate of return r satisfies

$$(1+r)^{10} = \frac{259.37 - 0.30(159.37)}{100} \quad \Leftrightarrow \quad r = 7.781\%.$$

3. (25 points total) We are given $r_f = 0.10$, $r_m = 0.18$, and $t_c = 45\%$.

(a) (8 points) Let us first calculate the expected return on TG's stock using the CAPM:

$$r_E = r_f + (r_m - r_f)\beta_E = 0.08 + (0.18 - 0.08)(1.5) = 0.23.$$

TG is financed with $100{,}000 \times \$12.50 = \$1{,}250{,}000$ of equity and \$750,000 of debt, i.e., $E = 1{,}250{,}000$, $D = 750{,}000$, $V = D + E = 2{,}000{,}000$, $D/V = 0.375$, and $E/V = 0.625$. This means that the present value of its tax shields is $t_c \times D = 0.45 \times 750{,}000 = 337{,}500$, and the value of its assets is

$$A = V - \text{PV(tax shields)} = 2{,}000{,}000 - 337{,}500 = 1{,}662{,}500.$$

In other words, a fraction $\frac{1{,}662{,}500}{2{,}000{,}000} = 0.83125$ of the firm's value comes from its assets, and a fraction $1 - 0.83125 = 0.16875$ comes from its tax shields. Since the firm's tax shields have the same risk as its debt, we have

$$\begin{aligned} & 0.83125r_A + 0.16875r_D = 0.375r_D + 0.625r_E \\ \Leftrightarrow \quad & 0.83125r_A + 0.16875(0.11) = 0.375(0.11) + 0.625(0.23) \\ \Leftrightarrow \quad & r_A = 0.2002. \end{aligned}$$

Since the project has the same risk as TG's assets, the appropriate discount rate is $r = r_A = 0.2002$.

Note that this rate could also have been calculated by unlevering TG's weighted average cost of capital (WACC). Indeed, TG's WACC is

$$\text{WACC}_L = (1-t_c)r_D\frac{D}{V} + r_E\frac{E}{V} = (1-0.45)(0.11)(0.375) + (0.23)(0.625) = 0.1664375.$$

Since $\text{WACC}_L = \text{WACC}_U\left(1 - t_c\frac{D}{V}\right)$, we have

$$r_A = \text{WACC}_U = \frac{\text{WACC}_L}{1 - t_c\frac{D}{V}} = \frac{0.1664375}{1 - (0.45)(0.375)} = 0.2002.$$

(b) (17 points) The adjusted present value (APV) of the project is given by

$$\begin{aligned} \text{APV} = -750{,}000 \quad & + \text{NPV(project)} \\ & + \text{PV(interest tax shields)} \\ & - \text{PV(after-tax issuance costs)}. \end{aligned}$$

In each of the project's five years, the after-tax cash flows will be

$$\begin{aligned} \text{CF} &= (\text{after-tax profits}) + (\text{depreciation tax shields}) \\ &= 360{,}000(1-t_c) + t_c\frac{750{,}000}{5} \\ &= 360{,}000(1-0.45) + 0.45(150{,}000) \\ &= 265{,}500. \end{aligned}$$

Therefore, using the discount rate r calculated in part (a), we have

$$\text{NPV(project)} = 265{,}500a_{5|0.2002} = \frac{265{,}500}{0.2002}\left[1 - \frac{1}{(1.2002)^5}\right] = 793{,}613.59.$$

Note that it could also be argued that the depreciation tax shields should be discounted at a rate lower than r, since they are not directly part of the project. For example, we could discount them at $r_D = 10\%$, since the company will benefit from these tax shields in the same years that it will benefit from the interest tax shields. They are also often discounted at $r_f = 8\%$ in practice. In the first case you would then get

$$\text{NPV(project)} = 360{,}000(1-0.45)a_{5|0.2002} + 0.45(150{,}000)a_{5|0.10} = 847{,}725.53.$$

In the second case, you would get

$$\text{NPV(project)} = 360{,}000(1-0.45)a_{5|0.2002} + 0.45(150{,}000)a_{5|0.08} = 861{,}355.35.$$

The present value of the interest tax shields can be calculated using the following table:

Year	Debt outstanding at start of year	Interest	Interest tax shield
1	750,000	75,000	33,750
2	600,000	60,000	27,000
3	450,000	45,000	20,250
4	300,000	30,000	13,500
5	150,000	15,000	6,750

Therefore, we have

$$\text{PV(interest tax shields)} = \frac{33{,}750}{1.10} + \cdots + \frac{6{,}750}{(1.10)^5} = 81{,}621.89.$$

Finally, the after-tax issuance costs are

$$\text{PV(after-tax issuance costs)} = 4\% \times 750{,}000 \times (1-0.45) = 16{,}500.$$

The adjusted present value of the project is therefore

$$\text{APV} = -750{,}000 + 793{,}613.59 + 81{,}621.89 - 16{,}500 = 108{,}735.48.$$

4. (20 points total) We are given $\beta_s = 0.75$, $r_f = 0.105$, and $r_m = 0.165$.

   (a) (4 points) Using the CAPM, the expected return on the stock is

   $$r_s = r_f + (r_m - r_f)\beta_s = 0.105 + (0.165 - 0.105)(0.75) = 0.15.$$

   We can find S_0, the current price of the stock, by discounting the expected cash flow from the stock at r_s:

   $$S_0 = \frac{290(0.5) + 170(0.5)}{1.15} = 200.$$

   (b) (6 points) In the up state, the option will be exercised and will pay $\$290 - \$212 = \$78$. In the down state, the option will not be exercised; the payoff is therefore zero. Let us form a portfolio by buying Δ shares of DJ's stock, and by lending $\$B$. In the up (down) state, this portfolio pays $290\Delta + 1.105B$ ($170\Delta + 1.105B$). We want

   $$\begin{aligned} 290\Delta + 1.105B &= 78; \\ 170\Delta + 1.105B &= 0. \end{aligned}$$

   Solving for Δ and B, we find $\Delta = 0.65$ and $B = -100$. This means that the call option can be replicated by buying 0.65 shares of the stock, and borrowing \$100.

   (c) (3 points) Since the portfolio has exactly the same payoff as the call option, its cost ($200\Delta + B$) should be the price of the call option in a well-functioning market:

   $$C_0 = 200\Delta + B = 200(0.65) + (-100) = 30.$$

   (d) (4 points) Since the call option is a portfolio of the stock and riskfree borrowing, the beta of the call option, β_c, is given by

   $$\begin{aligned} \beta_c &= \frac{200(0.65)}{30}\beta_s + \frac{-100}{30}\beta_{r_f} \\ &= \frac{200(0.65)}{30}(0.75) + \frac{-100}{30}(0) \\ &= 3.25. \end{aligned}$$

   The expected return r_c on the call option can then be obtained using the CAPM:

   $$r_c = r_f + (r_m - r_f)\beta_c = 0.105 + (0.165 - 0.105)(3.25) = 0.30.$$

   (e) (3 points) The expected payoff of the call option is $78(0.5) + 0(0.5) = 39$. The price of the option should therefore be

   $$C_0 = \frac{39}{1 + r_c} = \frac{39}{1.30} = 30.$$