QUESTION 1

Short Questions. This part consists of three short, stand-alone questions of lower math intensity. Please provide an answer or check the box corresponding to the correct answer (you are not required to show supporting work for this section).

A. Cleaning Service. Center City Cleaners offers office-cleaning services. Their costs (in dollars per hour) can be summarized by the graphs below. Assume that the market for office cleaning in Center City is perfectly competitive (therefore anyone can enter).

i. What is the lowest price above which Center City cleaners make a profit?

Price: ________________________________

ii. Between which two prices per hour would Center City Cleaners continue to produce in the short run, but shut down once there are no longer any fixed costs?

Prices: ____________________ and ____________________

iii. Below which price per hour would Center City Cleaners shut down even in the short run?

Price: ________________________________
B. **Electricity.** A recent government study finds that the production of electricity causes a negative externality of $0.10 per kilowatt hour (increased levels of acid rain reduce public health and destroy local crops). This is depicted in the graph below.

![Graph](image)

i. In general, if a good imposes a negative externality on society, _____ will be produced in the absence of intervention. If a good imposes a positive externality on society, _____ will be produced in the absence of intervention.

- Too much; too much _____
- Too much; too little _____
- Too little; too much _____
- Too little; too little _____

ii. Write the social welfare (or surplus) generated by this market with no intervention in terms of the labeled regions on the graph above (For example, A+B):

Social Welfare: ________________________________

iii. What would be the optimal tax per kilowatt hour for the government to levy?

Tax: _______________________________________

iv. Write the social welfare (or surplus) generated by this market if the optimal tax were levied in terms of the labeled regions.

Social Welfare: _______________________________
C. Auctions

i. For each of the following auction types, indicate whether someone making an optimal bid decides to “shade” (i.e. bid lower than their perceived valuation).

Private Value Second-price Sealed Bid Auction
   _____ Shade
   _____ Don’t Shade

Common Value Second-price Sealed Bid Auction
   _____ Shade
   _____ Don’t Shade

ii. If an auctioneer is choosing a format for a sealed-bid auction, and the same number of bidders will enter no matter the format, which one will have higher expected revenue?
   _____ First price
   _____ Second price
   _____ They will be the same

iii. You are entering a first-priced, sealed bid auction for pay-per-click advertising. You have some estimates on the probability of winning at each bid (represented as a payment per click) based on past auctions. You value each click at $0.70. Use the table below to select the optimal bid.

<table>
<thead>
<tr>
<th>Optimal?</th>
<th>Bid (per click payment)</th>
<th>Probability of winning</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$0.75</td>
<td>100%</td>
</tr>
<tr>
<td></td>
<td>$0.70</td>
<td>90%</td>
</tr>
<tr>
<td></td>
<td>$0.65</td>
<td>70%</td>
</tr>
<tr>
<td></td>
<td>$0.55</td>
<td>40%</td>
</tr>
<tr>
<td></td>
<td>$0.50</td>
<td>10%</td>
</tr>
<tr>
<td></td>
<td>$0.45</td>
<td>0%</td>
</tr>
</tbody>
</table>
QUESTION 2

Apple watch. Apple has recently introduced a new marquee product, the Apple Watch. Before they launched, they used focus groups with beta testers to estimate a demand curve, in order to decide on pricing. They estimated daily market demand for the Apple watch as:

\[ P = 900 - \frac{1}{2} q \]

Apple’s total cost to produce \( q \) units of Apple watch is:

\[ TC (q) = 108q + 0.1q^2 \]

A. What price should Apple set to maximize profits?

\[ Price: P = \] 

B. What is the profit that Apple will earn at this price?

\[ Profit: \] 

C. What is the consumer surplus created at this price?

\[ Consumer Surplus: \] 

Remember that Apple had estimated its demand curve from beta tester reports. Now, it wants to do a potentially more accurate job using real data. So, Apple sends 5% off coupons to a randomly selected group of potential customers and finds that it sells 15% more units of Apple watch to customers who get the discount.

D. Using this experiment, what is the own-price elasticity of demand for the Apple Watch?

\[ Elasticity \ of \ Demand: \] 

E. Assume that the coupon experiment represents a more accurate picture of demand than the beta tester reports. What should Apple do with the price charged in Part A?

_____ DO NOT CHANGE
_____ RAISE
_____ LOWER

Why?
QUESTION 3

**New Release.** Paramount Studios has struck a deal with Netflix to allow the streaming service to show its new movie line-up through a “New Release” channel. “New Release” would be available as a premium subscription service, where Paramount would license the content to Netflix, and Netflix would resell it to customers at a price of $p$ dollars per subscriber per month. Beyond payment to Paramount for the content, Netflix does not incur any additional distribution costs. Additionally, Paramount’s marginal cost for providing the movies to Netflix is zero, as in this case the movies are already made and streaming-ready.

The companies agree that the monthly market demand for the “New Release” channel is:

\[ q^D(p) = 15 - \frac{1}{3}p \]

where \( q \) is in millions of subscribers.

Under one possible payment structure, Paramount would license its content to Netflix for a monthly charge, called a “carriage charge,” of \( c \) dollars per subscriber per month.

**A.** In terms of the carriage charge \( c \), what is Netflix’s profit maximizing number of subscribers and price for “New Release”?

Subscribers: ___________________________

Price: ___________________________

**B.** What is Paramount’s optimal per-subscriber carriage charge? How much profit will Paramount make from “New Release” per month?

Carriage Charge: ___________________________

Profit: ___________________________

**C.** Given the carriage charge set by Paramount in B., what price will Netflix charge customers for “New Release”? How much profit does Netflix earn per month?

Price: ___________________________

Profit: ___________________________

Instead, Paramount decides that it would only be willing to sell content to “New Release” if Netflix pays a flat monthly fee, \( F \) (independent of the number of subscribers) and a per-subscriber carriage charge, \( c \).

**D.** How should Paramount set \( c \) to maximize joint profit, and what will be the total profit between both companies?

Optimal carriage charge \( c \): ___________________________

Profit: ___________________________
E. What is the range of possible values of \( F \), the fee paid by Netflix to Paramount, where both Netflix and Paramount would be willing to sign on to this new pricing scheme, and launch “New Release”?

\[ \underline{\text{______________}} > F > \underline{\text{______________}} \]

**QUESTION 4**

**T-shirts.** Everlane has cut through the middleman by taking t-shirts direct from factories to consumers. These t-shirt factories produce t-shirts using labor \((L)\) and fabric \((F)\). At a particular factory, a new technique of cutting fabric allows for less fabric waste if more labor is put in. Combining \(L\) units of labor and \(F\) units of fabric produces t-shirts according to the following production function.

\[
q = f(L,F) = 4 \times L^{0.5} F^{0.5}
\]

\(L\) costs $80/unit while \(F\) costs $20/unit.

A. Compute the marginal productivity of labor and fabric.

\[
MP_L = \underline{\text{___________________________}} \\
MP_F = \underline{\text{___________________________}}
\]

B. If this factory produces optimally, how many units of \(F\) should be used for every unit of \(L\)?

C. Suppose the manager of this factory wants to produce \(q\) t-shirts. How many units of \(L\) and \(F\) should be used?

(Note that both of these answers should be in terms of \(q\))

\[
L = \underline{\text{___________________________}} \\
F = \underline{\text{___________________________}}
\]

In general, the market for t-shirts is perfectly competitive with a large number of small factories each operating with a variable cost of

\[
VC(q) = 0.25 \times q^2
\]

for producing \(q\) t-shirts. Throughout, let \(p\) denote the price per t-shirt.

D. In this perfectly competitive environment, at a given price \(p\), how many t-shirts would a factory optimally choose to produce?

\[
q^* = \underline{\text{___________________________}}
\]

This is the factory’s supply curve, \(q^S(p)\).
Assuming that there are 100 such factories (all with identical cost structures) in the market, what is the market supply of t-shirts, \( Q^S(p) \)?

*Market supply curve: \( Q = \) ________________

Daily demand for t-shirts in the market is given by:

\[ Q^D(p) = 12,000 - 2,200p \]

E. What is the equilibrium price and daily quantity of t-shirts sold?

*Equilibrium price: ________________________________

*Quantity: ________________________________

Let’s say all t-shirts are currently produced in China, but many manufacturers are considering leaving for Vietnam, due to cheaper labor costs. As a result, the Chinese government offers a subsidy *(a negative tax)* of $0.60 per t-shirt produced domestically.

F. Find an individual factory’s supply curve once the subsidy takes effect. Then find the total market supply curve under the subsidy program.

*Firm supply curve: \( q = \) ________________

*Market supply curve: \( Q = \) ________________

What would the equilibrium price and daily quantity of t-shirts sold be now?

*Equilibrium price: ________________________________

*Quantity: ________________________________

**QUESTION 5**

**Rock Band.** A famous rock band is getting back together for a one-night-only event. They are allowing two local theaters, Roxy Theater and Rave Cinemas, to simulcast the performance.

The owners of both theaters estimate the market for simulcast concert tickets is a total of 1,000 people in the town. Each theater can seat at most 800 people and wants to maximize revenue (they’ve each already paid to simulcast the performance and have zero marginal costs, so maximizing revenue is maximizing profit).

The two theaters both have to announce their prices simultaneously on Ticketmaster on the same day. The pricing structure of both theaters only allows them to charge either $18, the regular ticket price, or $20, the special event price.

If they charge the same price, 500 people will see the performance at each theater. If they charge different prices, 800 will go to the cheaper theater, and the remaining 200 will go to the more expensive one.
A. Draw a game board for the two theaters (Roxy and Rave).

B. Do theaters have dominant strategies?
   
   _____ No
   _____ Yes

   If yes, the dominant strategies are:

   Find all the Nash Equilibria of the game.

   Nash Equilibria: ________________________________

In a surprise announcement, the band announces they will simulcast four more concerts over the next four months, and then they will definitely retire the simulcast program. Again, both theaters expect 1,000 fans each month.

C. What price per ticket do the theaters charge for the performance in each month if they play the Nash Equilibrium strategy suggested by game theory?

   Month 1: ________________________________
   Month 2: ________________________________
   Month 3: ________________________________
   Month 4: ________________________________

Now, in an even bigger surprise announcement, the artists announce that they plan to simulcast one new special performance each month from now on (i.e. indefinitely), starting today. As before, if the theaters charge the same price, 500 people will see the performance at each theater; if they charge different prices, 800 will go to the cheaper theater and the remaining 200 will go to the more expensive one. Both theaters discount future revenue at a monthly rate equal to $\delta$ (meaning, $1 received one month from now is worth $1/(1+\delta)$ today. Assume that revenue from ticket sales is received at the beginning of each month.)

D. What is each theater’s present value from always choosing the special event price (i.e. $20) forever?

E. What is the largest value of the discount rate $\delta$ such that “grim” trigger strategies can be used to sustain a Nash Equilibrium in which the theaters charge the larger price $20 rather than the lower price of $18?

   $\delta = ________________________________
QUESTION 6

**British Broadcasting.** In 1988, British Satellite Broadcasting (BSB) saw untapped potential in the UK TV market and entered with a strategy that centered on a high-tech infrastructure. Unfortunately for BSB, Rupert Murdoch’s media company, Sky Television, surprised BSB by considering its own entry with a less known, lower cost and lower quality satellite technology. We can model this with a sequential game in which Sky chooses to enter in 1989, BSB decides whether to exit in 1990, and then Sky decides whether to stay and fight if BSB does not exit. Payoffs are shown in the game tree below:

![Game Tree](image)

A. What are the equilibrium decisions of this game?
   - Sky: _____ STAY in ’89, STAY in ’90 _____STAY in ’89, EXIT in ’90 _____EXIT in ’89
   - BSB:_____ STAY in ’90 _____ EXIT in ’90

B. As a game-changing move, Sky is considering paying BSB to choose “exit”. How much would Sky have to pay to BSB for them to be willing to choose “exit” in 1990?
   - Sky would have to pay: _____________________________
   
   70M makes them exactly indifferent, so depending on what students assume (implicitly) the correct answer can range from 70M to 71M since any value in between will break the indifference.

C. By how much would Sky’s profit change from original outcome if it chose to pursue the buyout strategy from the previous part?
   - Sky’s profit changes by: _____________________________

EPILOGUE: Sky bought BSB in 1990 and sold it for scrap, but called the arrangement a “merger”. This is the origin of the current British television operator BSkyB.
Sample Exam Answers MGEC 611

QUESTION 1

A. i. 20
   ii. 10 and 20
   iii. 10

B. i. Too much; too little
   ii. A-D
   iii. $0.10
   iv. A

C. i. Private Value Second-price Sealed Bid Auction: Don’t Shade
      Common Value Second-price Sealed Bid Auction: Shade
   ii. They will be the same
   iii. $0.55 40%

QUESTION 2

A. 570
   The revenue function is
   \[ R(q) = p(q) \cdot q = (900 - 0.5q)q \]
   Take derivative w.r.t. q and MR is:
   \[ MR(q) = 900 - q \]
   Take derivative of TC w.r.t. q and MC is:
   \[ MC(q) = 108 + 0.2q \]
   At optimal \( q^* \), \( MR(q^*) = MC(q^*) \), so we get
   The equilibrium price is
   \[ 900 - q^* = 108 + 0.2q^* \]
   \[ q^* = 660 \]
   \[ p^* = 900 - 0.5 \cdot 660 = 570 \]

B. 261,360
   \[ \Pi^* = R(q^*) - TC(q^*) = 570 \cdot 660 - (108 \cdot 660 + 0.1 \cdot 660^2) = 261,360 \]

C. 108,900
   \[ CS = 0.5 \cdot q^* \cdot (\text{demand's price intercept} - p^*) \]
   \[ = 0.5 \cdot 660 \cdot (900-570) = 108,900 \]

D. -3
   \[ \varepsilon_d = \frac{q^*}{p^*} = -\frac{15}{5} = -3 \]
E. LOWER
To make the inverse elasticity pricing rule hold—right now markup is higher than the inverse elasticity, so markup should be lowered by lowering price

QUESTION 3

A. Subscribers: \(\frac{(45-c)}{6}\)
Price: \(\frac{(45+c)}{6}\)

Netflix profit function
\[ \Pi_N = (p - c)q(p) = (p - c)(15 - \frac{1}{3}p) \]
Take derivative w.r.t. p and set to 0:
The demand is:
\[ p^*(c) = \frac{(45-c)}{2} \]
\[ q^*(c) = 15 - \frac{1}{3}p^*(c) = \frac{(45-c)}{6} \]
Plug \(p^*(c)\) and \(q^*(c)\) back into the profit function: \(\Pi_N(c) = \frac{(45-c)^2}{12}\)

B. Carriage Charge: 22.5
Profit: 84.375

Paramount profit function: \(\Pi_P = cq(c) = \frac{c(45-c)}{6}\)
Use the FOC: \(c^* = 22.5\)
Then, \(q^* = q^*(c = 22.5) = 3.75\)
The profit is: \(\Pi_P^* = 22.5 \times 3.75 = 84.375\)

C. Price: 33.75
Profit: 42.1875

\[ p^* = p^*(c = 22.5) = \frac{(45+22.5)}{2} = 33.75 \]
\[ \Pi_N^* = \Pi_N(c = 22.5) = \frac{(45-22.5)^2}{12} = 42.1875 \]

D. Optimal carriage charge \(c\): 0
Profit: 168.75

The joint profit is maximized by setting the carriage charge to Paramount’s MC which is 0.
Plug \(c = 0\) into Netflix profit function:
\(\Pi_N(c = 0) = \frac{45}{12} = 168.75\)
So Netflix earns 168.75 − \(F\), and Paramount earns \(F\). The total profit is 168.75.

E. 126.5625 > \(F\) > 84.375
Netflix agrees if 168.75 − \(F\) ≥ 42.1875, so \(F\) ≤ 126.5625. Paramount agrees if \(F\) ≥ 84.375.
**QUESTION 4**

A. \[ MP_L = 2L^{0.5}F^{0.5} \]
\[ MP_F = 2L^{0.5}F^{0.5} \]

B. \[ \frac{2L^{0.5}F^{0.5}}{80} = \frac{2L^{0.5}F^{0.5}}{20} \]
\[ F = 4L \]

C. \[ L = \frac{q}{8} \]
\[ F = \frac{q}{2} \]
\[ q = 4L^{0.5}(4L)^{0.5} = 8L \]
\[ L = \frac{q}{8} \]
\[ F = 4L = \frac{q}{2} \]

D. \[ q^* = 2p \]
\[ MC(q) = \frac{d}{dq} VC = 0.5q \]
A competitive firm chooses quantity such that \( p = MC(q) \):
\[ p = 0.5q \]
\[ q^S(p) = 2p + 1.2 \]

E. **Equilibrium price:** 5
**Quantity:** 1,000

\[ Q^S(p) = Q^D(p) \]
\[ 200p = 12,000 - 2,200p \]
\[ p^* = 5 \]
So the equilibrium quantity is:
\[ Q^* = 200 \times 5 = 1,000 \]

F. **Firm supply curve:** \( q = 2p + 1.2 \)
**Market supply curve:** \( Q = 200p + 120 \)

Each firm’s marginal cost is reduced by the subsidy amount:
\[ MC(q) = 0.5q - 0.6 \]
Again, using \( p = MC(q) \),
\[ p = 0.5q - 0.6 \]
\[ q^S(p) = 2p + 1.2 \]
The market supply is:
\[ Q^S(p) = 100q^S(p) = 200p + 120 \]
Equilibrium price: 4.95
Quantity: 1,110

The equilibrium price is:
Q^S (p) = Q^D (p)
200p + 120 = 12,000 − 2,200p
p^* = 4.95
The equilibrium quantity is:
Q^* = 200 * 4.95 + 120 = 1,110

**QUESTION 5**

A.  

<table>
<thead>
<tr>
<th></th>
<th>Roxy</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rave</td>
<td></td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>9,000; 9,000</td>
<td>14,000; 4,000</td>
</tr>
<tr>
<td>20</td>
<td>4,000; 14,400</td>
<td>10,000; 10,000</td>
</tr>
</tbody>
</table>

If both price at 18, then each earns 18 * 500 = 9000. If posted prices differ, the one with 18 earns 18 * 800 = 14400, and the one with 20 earns 20 * 200 = 4000. If both price at 20, then each earns 20 * 500 = 10000.

B. Yes
If yes, the dominant strategies are: (18,18)
Nash Equilibria: 18,18

C. Month 1: 18,18
   Month 2: 18,18
   Month 3: 18,18
   Month 4: 18,18

D. \[10,000 * \left(1 + \delta\right) / \delta\]

E. \[\delta = 5/22 = 0.2273\]

\[10,000 * \left(1 + \delta\right) / \delta \geq 14,400 + 9,000 * \frac{1}{\delta}\]
\[\delta \leq \frac{10,000}{4,400} = \frac{5}{22}\]
**QUESTION 6**

A. What are the equilibrium decisions of this game?
   - Sky: EXIT in ’89
   - BSB: STAY in ’90

B. Sky would have to pay: 70M (or 70.01 or 71M, etc)

C. Sky’s profit changes by: 20 (19.99 or 19M etc.)